Mathemagical Card Tricks

Stephanie A. Santorico, Ph.D.
University of Colorado Denver
Department of Mathematical and Statistical Sciences
E-mail: Stephanie.Santorico@ucdenver.edu
Website: http://math.ucdenver.edu/~ssantori

Abstract:

There are many card tricks based on simple mathematics as opposed to sleight of hand. Here, we'll play with a number of such tricks, test them out and work on discovering the math underneath. For each trick, pair up and try it out, getting comfortable with the mechanics of the trick. Be sure to switch roles. Once you're comfortable with the trick itself, try to formalize the mathematics that makes the trick work. Tricks covered consist of:

- ***PICK A CARD***
- ***THE "FITCH CHENEY" TRICK***
- ***PICK-A-PAIR***
- ***PIANO DUET***
- ***MENTAL AGILITY***
- ***ANY WAY YOU COUNT THEM***
- ***RISING CARD (This is a fancy variation of the Pick-a-card trick)***
- ***THE O'HENRY TRICK***
- ***IMPOSSIBLE CARD LOCATION***

Acknowledgements:

Some of the material has been recycled from Tom Davis' lesson on "Mathematical Card Tricks" available at http://www.geometer.org/mathcircles/CardTricks.pdf. In addition, tricks were obtained from http://www.cardtricksite.com//tricks/math.htm and http://www.card-trick.com/mathematical_card_tricks.htm.

PICK A CARD

Deal 27 cards, face up onto a table in three columns with 9 cards in each. Ask your "victim" to choose a card but not to tell you what it is. Three times, ask in which column the card lies, and each time, collect the cards, putting the selected column between the cards in the other two columns, leaving the columns in order, and then deal the cards again by rows to make the next set of columns. After the third choice has been made and the cards re-dealt for the fourth time, the selected card will be number 14 – exactly half-way through the deck.

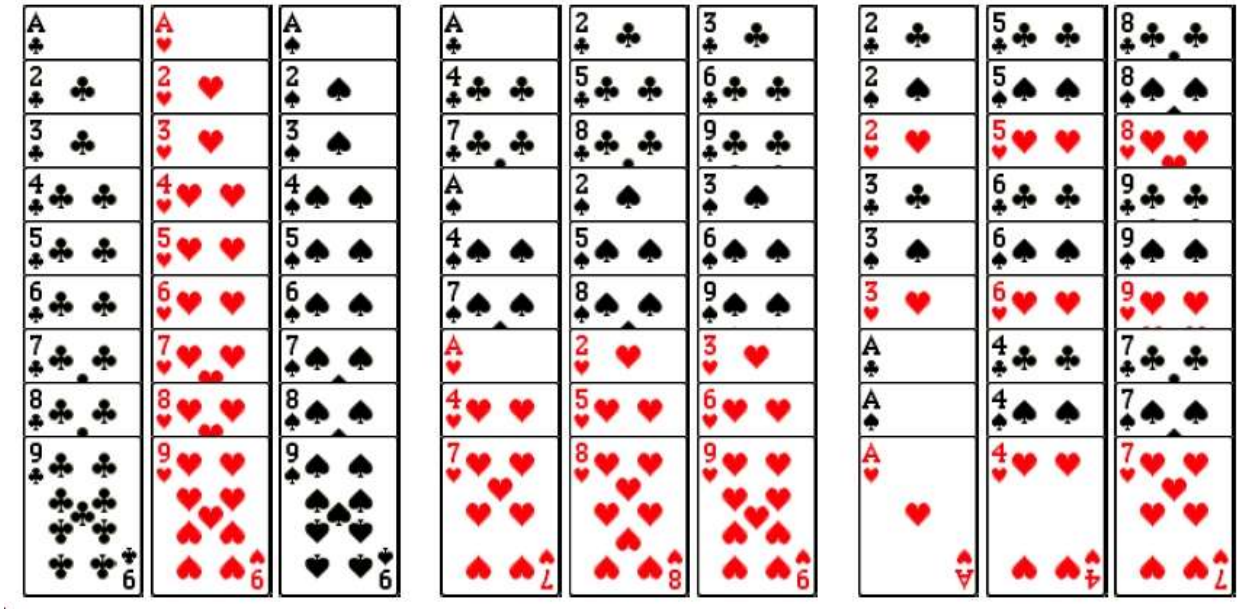
Extensions:

- Can you predict where the selected card will end up if you keep track of where the column is stacked each of the three times? For example, if the procedure is followed and the stack is placed first on the bottom, then at the top and finally in the middle, where will the card end up on the fourth deal?

- Can the same trick be performed with 3 stacks of 5? 5 stacks of 5? Etc.

A SOLUTION FOR THE PICK A CARD TRICK:

Each choice by the "victim" reduces the number of possibilities by 1/3. Since there were 27 choices originally, there will be 9 after the first selection, 3 after the second, and 1 after the third. You can show what is happening by starting with each column consisting of a different suit, and it's even better if the cards in the columns are in order, say A, 2, 3, …, 9. After the first selection, the selected suit must be in the middle three rows. The next selection will make the selected card be one of those in the middle row, and the final selection will force it to be in the exact middle: card number 14.



The three deals above show what happens if you begin with the cards in the arrangement on the left, and the "victim's" card happens to be a spade. The club row is put on top of the spade row and that on top of the heart row, so the second arrangement shows the cards after the second deal. All spades are in the middle three rows, in order.

Next, assume that the "victim's" card is in the third column (and so it must be a 3, 6 or 9, and we also know it was a spade). That column is put between the other two and dealt out, and as you can see, the 3, 6 and 9 of spades are in the center positions of each column. When the victim chooses his final column, and it is put between the other two, it will be the fifth card in the middle column, and will thus have 9 + 4 = 13 cards on top of it, making it the fourteenth card.

Note that after the third column selection is made, there is no need to re-deal the cards; you know the selected card is in the middle of the selected column, and once you know the card, you can do anything else in the world with the deck: shuffle it, add more cards from the rest of the deck, et cetera. Note also that if you make different, but consistent, choices of where to insert the selected column, you can make the chosen card appear in any position. For example, if you place the selected column on top each time, after the third choice, the card will be on the very top of the pile.

Extensions:

- Consider the cards organized in three blocks of three rows with three columns. If you can keep track of where the stack is placed first (F), second (S) and third (T), then on the fourth deal the card will be located at the T block, S row and F column.

THE "FITCH CHENEY" TRICK

The "Fitch Cheney" trick is fairly sophisticated. It requires an accomplice who is also good. The victim, however, need not be sophisticated at all.

The victim selects any five cards from a deck of cards while the performer is not present and gives them to the accomplice. The accomplice picks a single card from the set, places it face-down, and hands the four remaining cards to the performer when she returns. The performer looks at the four cards and names the face-down card. In fact, the accomplice can give the pack of four cards to the victim to hand to the performer and leave the room to assure that he is not giving the performer any information by body-language, et cetera. All the performer has to work with is the deck of four cards.

On the face of it, this is impossible. The performer sees only four cards which could be arranged in $4! = 24$ different orders, but there are $52 - 4 = 48$ possibilities for the remaining face-down card. But there is more: the accomplice can not only arrange the deck of four cards, but can choose the face-down card, so he actually has $5 \times 4! = 120$ different courses of action. There are many ways to do this. Can you come up with a system that allows for complete determination of the fifth card?

POTENTIAL DISCUSSION FOR THE FITCH CHENEY TRICK:

What follows is a reasonable approach that does not require too much mental effort (although it does require some, and definitely requires practice, especially to do it quickly).

First, the accomplice and the performer need to agree on a ranking for a deck of cards. Any ranking will do, and if they are bridge players, for example, they might agree that any 2 is less than any 3, and so on, up to jack, queen, king, and ace is the "largest". If there are two equal cards, the "bridge ordering" of the suits determines the larger card:

clubs < diamonds < hearts < spades.

Since there are five cards, at least two of them must be in the same suit, and if you agree to count in a circle, as: 2, 3, 4, . . . , 9, 10, jack, queen, king, ace, 2, 3, . . . , then starting from one of those two cards you can get to the other in six or fewer steps. For example, if the two cards are the 4 and king, then the king is the starting card, and counting up as: "ace, 2, 3, 4" makes the 4 exactly four steps up from the king.

The accomplice places this starting card on top of the deck of four cards to be handed to the performer, so the performer immediately knows the suit and a starting place. To determine the card, she only needs to know how many steps up to take to get there, and this number will be between 1 and 6. Luckily, there are six permutations available for the remaining three cards, and the performer and accomplice need only agree on an assignment of orderings to numbers to pass this information using the remaining three cards. The three cards are ranked as discussed above, so assume that *A*, *B* and *C* represent the smallest, middle, and largest card, respectively, using that ranking. Here's a possible assignment:

1 = *ABC* 2 = *ACB* 3 = *BAC* 4 = *BCA* 5 = *CAB* 6 = *CBA*.

This is relatively easy to remember since the rank orders are in alphabetical order.
As an example, suppose the five cards chosen by the victim are 2*S*, 2*H*, *QH*, 5*D* and 2*C*, where "*QH*" means the "queen of hearts", et cetera.

The only pair of cards that are in the same suit are the 2*H* and the *QH*. It's too many steps from the two to the queen, so the queen of hearts must be the starting card, and there are three steps necessary to get to 6 the two from the queen: "king, ace, 2". Thus the accomplice places the queen of hearts on the top of the deck of four cards and places the two of hearts face-down on the table. The remaining cards are ordered as follows: 2*C* < 2*S* < 5*D* and we need to order them to encode "three steps". Three is encoded by "middle, low, high", or 2*S,* 2*C,* 5*D*, so the deck handed to the performer looks like: *QH,* 2*S,* 2*C,* 5*D*.

PICK-A-PAIR

Pre-arrangement : Remove all tens and picture cards from the deck.

The magician asks a spectator to shuffle the cards. He then takes a bunch of cards and fans them in front of the spectator who is asked to pick a card from the fan. The spectator is to remember the chosen card and replace it in the pack.

The magician then asks the spectator to multiply the value of the card they picked by 2 and then after adding 5, to multiply that number by 5. The spectator is then asked to remember the total they arrive at.

Next, the magician lets the helper select another card in the deck, and that value, they are to add to the total that they had arrived at earlier. The spectator is finally asked to reveal their total to the magician. At which point the magician reveals their the original two cards.

POTENTIAL DISCUSSION FOR PICK-A-PAIR TRICK:

When you hear their total, subtract 25. **The two digits you get will be the two cards they chose.**

For example, if their total is 76 then their two selected cards would have been an 8 and a six (86 – 25 = 51 -> cards were a 5 and a 1). This is a self working mathematical trick.

PIANO DUET

The magician invites a volunteer to sit at a his table and to place his hands as if playing the piano. He then places two cards between the third and little finger of the volunteers left hand saying: 'Here's one pair'. Then another two cards between the third and second fingers saying:'Here's another pair'. Next the magician puts two cards between the first and second fingers saying: 'And here's another pair' Lastly, he places two cards between the volunteers forefinger and thumb saying: 'And yet another pair here as well'.

The magician does the same with the volunteers right hand except this time he puts only one card between the thumb and forefinger stressing: 'but this one is an odd card'.

The performer repeats what he has done always stressing that cards were placed in pairs. He then continues in the same fashion only this time he removes each of the pairs of cards from the volunteers left hand, separates them and places them side by side on the table - again stressing: 'Here's a pair'. He continues with the helper's right hand pairs until he reaches the one odd card which he hands to the volunteer asking him to place it on top of either of the two piles.

This done, the magician taps the pile on which the volunteer placed the odd card and says that the odd card will magically fly across to the other pile. He then picks up the pile which is supposed to have the extra card added and separates the cards into side by side pairs saying each time once again: 'Here's a pair'.

The volunteer sees that there are four pairs of cards and the odd card has apparantly vanished! The magician separates the other pile in the same manner and the volunteer sees that there is an odd card! - the illusion is complete - the odd card must have jumped across to the other pile!

POTENTIAL DISCUSSION FOR THE PIANO DUET:

This trick works because of course earlier in the trick the volunteer is holding 4 pairs of cards in the left hand and 3 pairs and one odd card in the right. The cards are then divided into two piles of seven cards which the volunteer doesn't notice because of the emphasis on pairs. When the odd card is added to a pile it turns it into an even numbered pile!

MENTAL AGILITY

This mathematical trick can be performed with any pack. The magician can reveal a chosen card even when it is selected with his back turned.

Get a spectator to think of a number between one and 10. Then ask him to shuffle the pack (this can be be his own if you like) and get him to count down to the number thought of and make a note of the card but to leave it in the same position. Get him to do this while your back is turned.

After he has done this, turn around and take the pack placing it behind your back then rapidly count off 19 cards and as you do so reverse their order replacing them on the top of the pack. As you do this, say that you will put the card at number 20.

Finally, bring the pack from behind your back and ask the spectator which number he thought of -- for example if it was a five then begin your count with in this example 6 dealing the cards one at a time. When you reach 20, get the spectator to name his card and when you turn it over, to his amazement, it is his card.

VARIATION ON MENTAL AGILITY:

Whilst your back is turned, get a spectator to shuffle a pack of cards which can be his own if you like, then to select a card from the pack and place it face down on the table. Next, ask the spectator to make two small piles of cards each with the same even number next to his selected card (for example two piles of four cards or two piles of five cards) and to place one of the piles in his pocket and the other one on top of the selected card which is face down on the table. Finally ask the spectator to take the a pile of cards on the table including his selected card and placed the pile on top of the pack. After all this is done you can turn around.

Now you, the magician pick up the pack and put it behind your back reminding the spectator how you couldn't possibly know which card he selected and its position in the pack. Next count 15 cards from the top reversing their order as you do so and replace them on top of the pack. Bring the pack round the front until the spectator that you are now going to make the situation even more difficult for yourself by asking him to take the small packet of cards from his pocket and place them on top of the pack also. Get the spectator to do this and when it is done, the selected card will now be the 15th card from the top of the pack and of course there are many ways that you can reveal this in as entertaining way as possible.

ANY WAY YOU COUNT THEM

1. Tell a spectator that each card represents a number--A is 1, J is 11, Q is 12, K is 13, and the rest have their face value.

2. Tell the person to pick up the top card and place it face up on the table.

3. State its number and then count onto it enough cards to reach 13. For example, if the card is a Jack, say "Eleven..." and then deal two cards onto it while you say, "...twelve, thirteen."

4. Flip over this stack and set it aside.

5. Repeat Steps 2-4 until 13 cannot be reached with the remaining cards. Set these aside (if the count works out evenly, don't worry).

6. Have the spectator choose 3 piles. Remove all of the other piles and add them to the cards (if any) remaining from step 5.

7. Pick up the pile of cards remaining from steps 5-6, count off 10 cards and set them aside.

8. Have the spectator choose 2 of the remaining 3 piles. Turn the top cards on these 2 piles face up.

9. Add together the values of the two face-up cards, count out their sum from the cards you're holding and set them aside.

10. Have the spectator count the number of cards left in your hand.

11. Tell him or her to flip over the top card on the remaining pile. It is equal to the number of cards left in your hand.

RISING CARD (This is a fancy variation of the Pick-a-card trick)

1. Deal out five piles with five cards in each pile.

2. Tell the spectator to pick a pile. Pick up the pile and tell her (or him) to look at any specific card and memorize it.

3. Put the pile down.

4. Pick up two of the other piles. Put her pile on top of those two, and put the two remaining piles on top. Her pile will be the middle one.

5. Deal out the cards in the OPPOSITE order. That is, form the stacks by laying cards down for stack 1, then stack 2, then stack 3, then stack 1 etc.

6. Show her the cards in one pile and ask if she sees her card. Do this until you find her pile again.

7. Combine two of the piles into a single stack.

8. Pick up the top card of her pile and place it on the stack so that it projects about one inch beyond the end. Pick up the next card from her pile and place it on the other so that its end projects half an inch in the opposite direction. Continue with the other three cards in her pile so that the third card sticks out an inch at one end, the fourth card sticking out a half-inch at the other end, and so on.

9. Place the remaining two piles on top of the rest of the cards.

10. Hold up the deck so that three cards stick out of the top and two cards out of the bottom. Tap the three cards down until they're flush.

11. Tap the bottom two cards up until they're flush.

12. One card will rise above the rest. This is her card!!!!

THE O'HENRY TRICK

Preparation: Before the trick make up a packet containing 20 black cards and one red (we'll say the Queen of Hearts.) Put the red card 10th from the top. Place this packet on top of the rest of the pack.

In performing the trick say to the audience you will remove a random number of cards from the top of the pack, but remove the top 21 cards placed there earlier and disregard the rest of the pack (you no longer require them).

Give the packet of 21 cards to the spectator and turn your back. Tell the spectator to remove between one and ten cards from the top of the packet, count the number to themselves and put these cards into their pocket.

You tell the spectator you will predict which card they will choose and write it on a piece of paper. Write down the Queen of Hearts (or whatever the red card was), fold the paper and give it to a member of the audience to hold.

Turn back to face the spectator. Deal out from right to left ten cards from the remainder of the packet. Ask the spectator how many cards they have in their pocket and remind them that they had a free choice of between one and ten. Count from left to right the number they tell you. The last card counted is theirs (it's the Queen.) Open the piece of paper and show your prediction to be correct.

To top off the trick ask them to return the cards from their pocket and tell them that not only did they pick the card on the slip of paper, but they chose the only red card in the pack. Show that all the other cards were black.

IMPOSSIBLE CARD LOCATION

Using a 52-card deck, have three people each select a card without showing it to you. Tell them to memorize their card.

Deal one pile of 10 cards face down. Next to it deal a pile of 15 cards, and next to that deal another 15-card pile. Keep the remaining 9 cards in your hand. Have the first person put his (or her) card on top of the 10-card pile, cut as many cards as he wants from the second pile, and put them on his card. Have the second person put her card on the second pile, cut as many cards as she wants from the third pile, and put them on top of her card. Have the third person put his card on top of the third pile, hand him the 9 cards you're holding, and have him place them on top of his card.

Pick up the last pile, put it on the middle pile, and put both on the first pile. Make clear that the cards are now lost and you will find them. Take four cards off the top and place them on the bottom of the deck. Explain that you are going to flip a card up and next to it one down and keep on repeating this until you don't have cards in your hand. Tell the spectators to say "Stop" if they see their card. Deal the cards alternately into two piles, one face up and one face down, starting with the face-up pile. When all the cards have been dealt (the spectators won't see their card unless you mess up), push the face-up pile aside and pick up the other pile. Deal it into two piles in exactly the same way. Keep repeating this until you have only three cards left face down. Turn them over, and there are their cards. The top one is the third person's card, the next is the second person's card, and the bottom one is the first person's card.