

Grid Luck

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These problems should be done on grid paper - as many others, geometric and not. Thus the title of the talk.

1. (a) How many squares are there in a 2×3 rectangle? 5×8 rectangle? $m \times n$ rectangle?
(b) How many rectangles are there in a 2×3 rectangle? 5×8 rectangle? $m \times n$ rectangle?
2. On your grid paper, draw a rectangle and one of its diagonals. How many grid squares are crossed by the diagonal?
3. (a) How many points in the plane can be found with the property that the distance between any two of them is always an integer (not necessarily the same integer)?
(b) How many non-collinear points in the plane can be found which have the property that the distance between any two of them is an integer?
4. Draw a square and connect each vertex with the midpoints of two opposite sides (8 lines altogether). How many right triangles can you find in this diagram? How many of these triangles are 3-4-5 triangles? Can you prove that they indeed are of this type?
5. Given 2010 squares of different sides, is it possible to cut them in a number of pieces that can be rearranged to form one large square?
6. Find an infinitely long path in the plane starting at the origin and having the property that from any point in the plane one can reach the path by moving a total distance of less than one unit.
7. Draw a rectangle and trace the path of a billiard ball that begins in the lower left corner and initially travels upward at a 45-degree angle. Assuming that the ball bounces off the walls at perfect 45-degree angles, which corner does the ball reach first? What fraction of all the unit squares within your rectangle does the ball pass through on its way? Start your experiment with a rectangle having width 3 and height 5, then choose other dimensions. What will happen if the width is m and the height n units for some positive integers m and n ?
8. How many ways can you place n dots into an n by n grid so that no dots are in the same row or column?
9. Is it possible to dissect a square into 2 squares? 4 squares? 6 squares? 7 squares? Can you decide for which positive integers n it is possible to dissect a given square into n smaller squares?

10. And don't forget the Infinite Tic-Tac-Toe!