Problems to Tease and Teach

Tatiana Shubin (shubin@math.sjsu.edu)

The main unifying theme of the collection of problems below is that each one of them has an "Aha!" solution. Try your hand at finding these solutions. Then see what other links are between these problems, and between these problems and the topics you encounter in your classroom.

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1. (a) Five diggers can dig a trench five meters long in five days. How many diggers would dig a trench a hundred meters long in a hundred days?

   (b) Alice and Bob have to type up a manuscript. Alice can type the whole manuscript in two hours while Bob would take three hours to do the job. What is the shortest time they need to type the manuscript if they work together?

2. A jar filled with honey weighs 500 grams. The same jar full of gasoline weighs 350 grams. Honey weighs twice as much as gasoline. How much does the empty jar weigh?

3. The total weight of four cats and three kittens is 15 kilograms, while the total weight of three cats and four kittens is 13 kilograms. What is the weight of each cat and each kitten? (Assume that all cats have equal weight, as do all kittens.)

4. Two trains leave their stations, A and B, at the same time and move towards each other. After they meet, the first train reaches station B one hour later, while the second train reaches station A after two hours and 15 minutes. How much faster is the first train moving? (Assume that that each train maintains a constant speed.)

5. (*"Problems for Kids from 5 to 15", Vladimir I. Arnold*) At sunrise two old women started to walk towards each other. One started from point A and went towards point B while the other started at B and went towards A. They met at noon but did not stop; each one continued to walk maintaining her speed and direction. The first woman came to the point B at 4:00 pm, and the other one came to point A at 9:00 pm. At what time did sun rise that day?

6. (*"Aha! Solutions", Martin Erickson, to be published by the MAA*) A man is crossing a train trestle on foot. When he is 4/7 of the way across he sees a train coming toward him head-on. He realizes that he has just enough time either to run toward the train and get off the trestle or run away from the train and get off the trestle. If the man can run 20 kilometers per hour, how fast is the train going?

7. (*"Problems for Kids from 5 to 15", Vladimir I. Arnold*) Suppose we have a barrel of wine and a cup of tea. A teaspoon of wine is taken from the barrel and poured into the cup of tea. Then the same teaspoon of the mixture is taken from the cup and poured into

the barrel. Now the barrel contains some tea and the cup contains some wine. Which volume is larger – that of the tea in the wine barrel or of the wine in the teacup?
Note: The same question could be asked after the process has been repeated several times.

8. How high can we stack all the one cubic millimeter cubes that can fit in one cubic meter if we put them in a single stack?

9. During the storm last week, my house lost electricity. I lit two candles and worked on an interesting problem using candlelight until the electricity came back on. The next day I wanted to find out how long the blackout lasted. All I knew was that to begin with, both candles were of the same length but of different thickness – the thicker one would burn out completely in five hours while the thinner one would only last four hours. Unfortunately, I could not find the remaining pieces of the candles since my son threw them away – he said that there had been only little pieces left not worthy of keeping. All he remembered was that the stumps of the candles were of unequal lengths – one was four times longer than the other. How long did I use the candles?

10. A rancher split his cattle between his sons as follows. The eldest son got one cow and 1/7 of the remaining cows; the next son got two cows and 1/7 of the remaining cows; the third son got three cows and 1/7 of the remaining cows, and so on. In this way, the entire herd had been split up entirely between all of the sons. How many sons did the rancher have and how large was his herd?

11. Alice has 12 cookies and Bob has 9 cookies. Charley, who has no cookies, pays Alice and Bob 42 cents to share their cookies. Each one of them eats one-third of the cookies. Bob says that he and Alice should split 42 cents evenly, and Alice thinks that she should get 24 cents and Bob should get 18 cents. What is the fair division of 42 cents between Alice and Bob?

12. Two friends were cooking rice. One put 200 grams of rice in the pot and the other, 300 grams. When the rice was ready, a third friend came in and the three of them ate all the rice. When the third friend was leaving, he gave his friends $5 to pay for this share of the rice. How should the two friends split the money? (We assume that every person ate exactly 1/3 of the rice.)

13. (a) We want to split 9 apples evenly between 12 kids in such a way that not a single apple gets cut into more than four parts. How can this be done?
(b) Is it possible to split 5 apples evenly between 6 people without cutting a single apple into more than 3 parts?

14. Can you express the number 1 using each of the 10 digits exactly once?

15. Four people move along a road: one is driving a car, the second one is riding a motorcycle, the third one is riding a moped, and the fourth one, a bicycle. Each one maintains a constant speed. The car driver caught up with the moped at 12:00 pm, met the

bicyclist at 2:00 pm, and met with the motorcyclist at 4:00 pm. The motorcyclist met the moped at 5:00 pm, and caught up with the bicyclist at 6:00 pm. At what time did bicyclist meet the moped?

16. (*"Aha! Solutions", Martin Erickson, to be published by the MAA*) Seven particles travel up and down on a straight vertical line bounded by two endpoints. Initially, they all move upward with the same speed. When a particle strikes another particle, or reaches either endpoint, its direction of motion changes (while the speed remains constant). How many particle-particle collisions occur before the particles again occupy their original positions and are moving upward?

17. (*"Topics in the Theory of Numbers", Paul Erdös and János Surányi*) There is a plate of cherries on a table. In our absence, someone serves the cherries in the following way: for every 5 cherries taken from the plate, one is put on a second plate and the other four are put in a serving bowl. This continues until there are fewer than five left on the original plate. From the second plate the procedure continues, using a third plate until there are fewer than five cherries left on the second plate. This continues until there are fewer than five cherries on the last plate. The bowl is then put away. We are presented with only the plates and asked to determine how many cherries there were at the beginning. How do we do this? How many cherries were there originally if on 4 plates we now have 2, 4, 0, and 3 cherries, in this order? How would we do it if instead of groups of five, the cherries were served in groups of two or groups of ten (one cherry placed on the new plate, the remaining placed in the bowl)?