Problems for Austin Teachers Math Circle, November 20, 2014
Hats and More!

1. Three prisoners will be given red or blue hats (chosen at random), and arranged so that they can see the other two prisoners' hats, but not their own. They all must simultaneously guess their own hat color. If all three get the answer right, they will be freed. If even one of them gets his color wrong, something horrible will happen to them. (You can supply the details.)

   They are allowed to discuss the problem ahead of time and make a strategy, but there is no possible communication once the hats are revealed. What strategy maximizes their chance of freedom, and what is that probability?

   [If you get stuck, try the analogous problem with 2 prisoners]

2. Same problem, only with N prisoners instead of just 3.

3. 3) Now you have N prisoners, arranged in a line. Each prisoner can see the hats of the prisoners in front of him, but not his own or the ones behind. Starting at the back, each prisoner guesses his hat color. If he gets it right, he is freed. If not, he is punished (again, supply the gory details yourself). As before, the prisoners can decide on their strategy ahead of time. What strategy will minimize the average number of prisoners who get punished? [You may want to start with $N = 2$ and work up.]

4. 4) Same setup of problem 1, except that a) the prisoners don't have to guess simultaneously (but they do have to guess within, say, one hour), b) when each person guesses, everybody else gets to hear the guess, and c) everybody is freed if ALL the guesses are right, but everybody is punished if even one person gets it wrong.

   Find a strategy with a high probability of getting everybody released.

5. 5) And now for something completely different. Let B be a convex body in the plane. Show that there is a way to cut B along a straight line so that each piece has the same area and the same perimeter.

6. 6) And another completely different problem. Show that some number of the form 99999...9 is divisible by 324,137. (And no, there is nothing special about the number 324,237.)