Perfect Shuffles

Number the locations in a deck by how many cards are above them:

0 1 2 3 4 5 ... n-1

In one perfect shuffle a card at location x in a deck of n cards is moved to location $2x \pmod{n-1}$

It is fairly easy to convince yourself of this. Cards in the top half of the deck are easy. If x cards are above them, x more are inserted above them when we do a shuffle.

So if we do k shuffles cards starting at location x end at location $2^k x \pmod{n-1}$

If $2^m \equiv 1 \pmod{n-1}$ then m perfect suffles returns a deck of n cards to their original arrangement.

It can happen that there is an exponent m and numbers x & y so that

$$2^m x = x \ , \quad 2^m y = y \pmod{n-1}$$

but $2^m \not\equiv 1 \pmod{n-1}$

Still, we can answer the "how many shuffles of n cards" question by finding the smallest positive number so that $2^m \equiv 1 \pmod{n-1}$

Example: repeated doubling modulo 51

$1, 2, 4, 8, 16, 32, 64 \equiv 13, 26, 52 \equiv 1$

Shows $2^8 \equiv 1 \mod 51$ so

8 perfect deals of a standard deck returns it to its original order.