PAMTC

Simplex Lock

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1 Asking Questions

Consider the simplex lock below.

What kinds of mathematical questions can we ask about this lock?

What is the combination? What is the probability of guessing the combination? How many different possible combinations are there? If we lost the combination, and wanted to go through all the possible combinations in some kind of systematic way until we found the correct one, how would we do so? How long would it take to go through all of the combinations systematically? How many possible combinations do we expect to have to try until we find the correct one?

2 Choosing a Question

Let's consider the question of how many different combinations are possible. What do we need to know about this lock to be able to answer this question?

Do we need to push all the buttons? Can we push two or more buttons at once? Can we push a single button more than one time? Does the order in which we push the buttons matter?

3 Defining the Problem

Version 1: How many possible combinations does a five-button simplex lock have if:

1. A combination is a sequence of 1 or more pushes, each push involving at least one button.
2. Each button may be used at most once (once you press it, it stays in).
3. Each button must be used at least once (there are no "open" buttons).
4. When two or more buttons are pushed at the same time, order doesn't matter.

Example: Suppose there are only two buttons. Then there three possible combinations: first 1 and then 2; first 2 and then 1; both 1 and 2 at the same time.

Version 2: How many possible combinations does a five-button simplex lock have if:

1. A combination is a sequence of 0 or more pushes, each push involving at least one button.
2. Each button may be used at most once (once you press it, it stays in).
3. Each push may include any number of "open" buttons, from one to five.
4. When two or more buttons are pushed at the same time, order doesn't matter.[1]

4 Useful Fact

The number of ways to choose k different items from a set of n distinct items is denoted $\binom{n}{k}$ and is equal to

$$\frac{n!}{k!(n-k)!}.$$

For example, the number of ways to choose $k = 3$ different letters from the set $\{a, b, c, d, e\}$ of $n = 5$ distinct letters is

$$\binom{5}{3} = \frac{5!}{3!(5-3)!} = \frac{5!}{3!2!} = \frac{5 \times 4 \times \cancel{3} \times \cancel{2} \times \cancel{1}}{\cancel{3} \times \cancel{2} \times \cancel{1} \times 2 \times 1} = \frac{5 \times 4}{2 \times 1} = 10.$$

The 10 choices are: $abc, abd, abe, acd, ace, ade, bcd, bce, bde, cde$.

[1] I found this problem in *Mathematical Connections: A Companion for Teachers and Others* by Al Cuoco.

5 Answers

Let $f(n)$ denote the number of combinations for an n-button simplex lock as described in Version 1. Let $g(n)$ denote the number of combinations for an n-button simplex lock as described in Version 2. Define $f(0) = 1$.

n	$f(n)$
0	1
1	1
2	3
3	13
4	75
5	541

n	$g(n)$
1	$\binom{1}{0}f(0) + \binom{1}{1}f(1) = 2$
2	$\binom{2}{0}f(0) + \binom{2}{1}f(1) + \binom{2}{2}f(2) = 6$
3	$\binom{3}{0}f(0) + \binom{3}{1}f(1) + \binom{3}{2}f(2) + \binom{3}{3}f(3) = 26$
4	$\binom{4}{0}f(0) + \binom{4}{4}f(1) + \binom{4}{2}f(2) + \binom{4}{3}f(3) + \binom{4}{4}f(4) = 151$
5	$\binom{5}{0}f(0) + \binom{5}{1}f(1) + \binom{5}{2}f(2) + \binom{5}{3}f(3) + \binom{5}{4}f(4) + \binom{5}{5}f(5) = 1082$

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