Magical Questions about Magic Squares and Magic Graphs

MAGIC SQUARES

A magic square is an $n \times n$ array whose entries are the integers $1, 2, \ldots, n^2$ where the sum of each row, column and the 2 main diagonals equals the same sum.

Question 1: Are there any 2×2 magic squares? Why or why not?

Question 2: How many 3×3 magic squares can you find?

Question 3: For a given $n \times n$ magic square what is the magic constant (i.e. the sum of each row/column/diagonal)?

Constructing Magic Squares of odd order:

1. Place a 1 in the middle entry of the top row.
2. Now we'll place the numbers 2 up to n^2 in consecutive order by using the following rules:
a. Place the next number diagonally up and to the right if that space is open. When you move out of the grid, wrap around top to bottom and right to left.
b. If that space is not open, return to your starting point and place the next number directly below your current number.
3. Continue this process.

Question 4: Why does this process work?

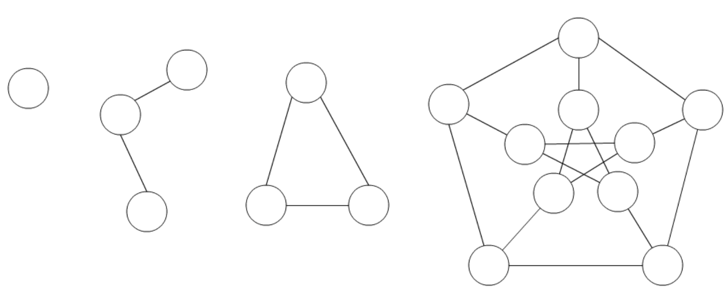
You can create many other types of problems by changing the conditions. How can we change the conditions and create new problems?
(Chapters 6-9 in Before Sudoku: The World of Magic Squares)

Magic Squares and art: Starting on p. 130

16	3	2	13
5	10	11	8
9	6	7	12
4	15	14	1

GRAPH THEORY

A *graph* is a collection of vertices (dots) and unordered pairs of vertices known as edges (lines).



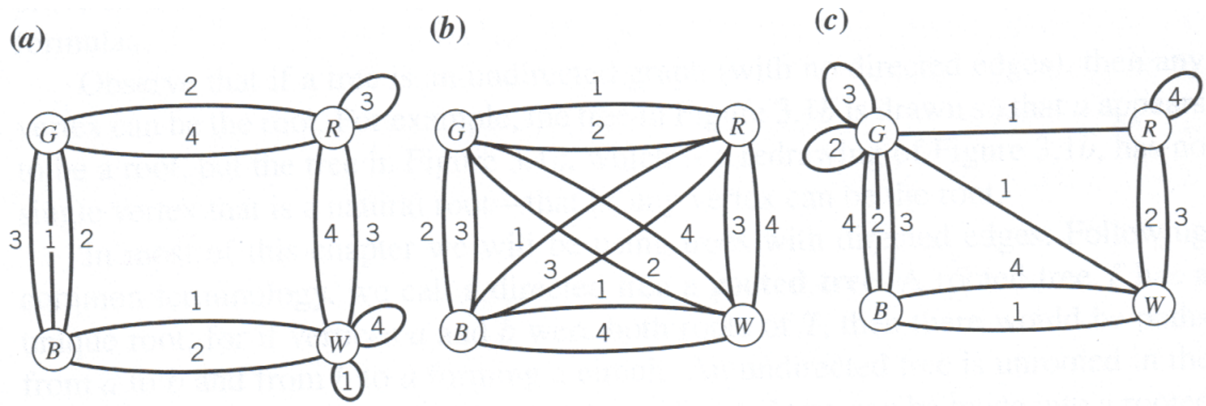
- Bridges of Konigsberg
- Line drawing puzzles

  `http://education.ti.com/calculators/downloads/US/Activities/Detail?id=6255`

- Utilities problem
- Many others!

  `http://www.teachforever.com/2009/11/we-still-use-math-every-day-using.html`

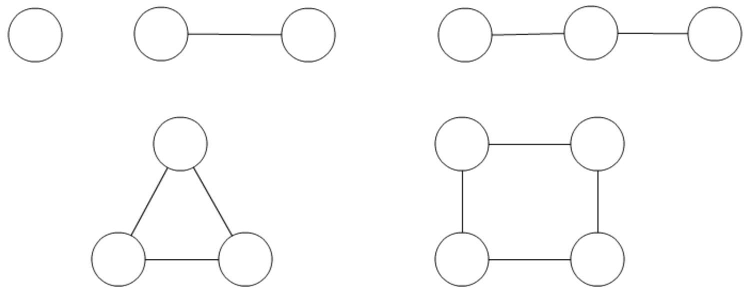
Instant Insanity
Question 5: Determine all the Instant Insanity solutions for the following puzzles. If there are none, explain why.



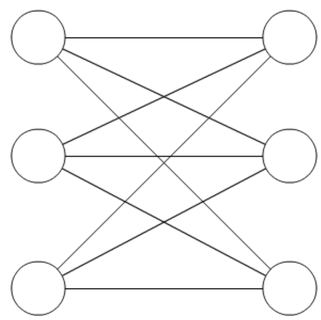
<u>MAGIC GRAPHS</u>
A *graph labeling* is an assignment of numbers to the vertices and/or edges of a graph such that certain conditions hold.

(VERTEX)-MAGIC LABELING: Place distinct numbers on the edges so that if you take any vertex and add all the labels touching that vertex you get the same sum at EVERY vertex.

Question 6: Which of the following graphs have a magic labeling?

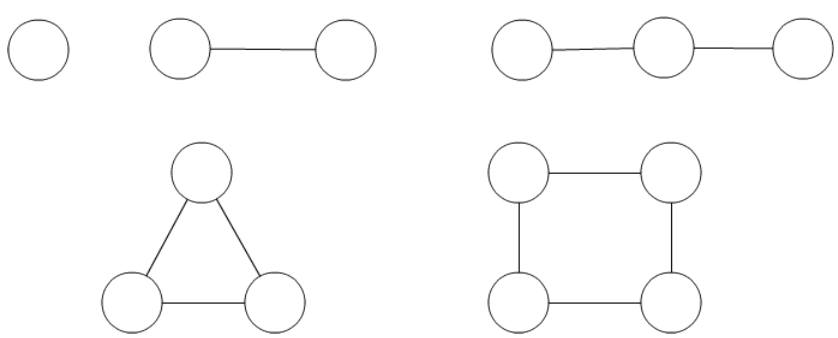


Problem 7: Find a magic labeling for the following graph:

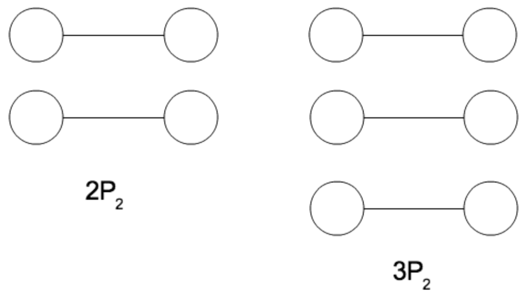


EDGE-MAGIC TOTAL LABELING: Place the numbers 1 up to $v + e$ on the vertices AND edges so that each number is used exactly once and that if you take any edge in the graph and add the label on that edge plus the labels on each end vertex you get the same (magic) constant k.

Question 8: Which of the following graphs have an edge-magic total labeling?



Question 9: When does nP_2 have an edge-magic total labeling? Consider n even and n odd.



Open Question: Which flowers and helms are edge-magic?

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