**MATH TEACHERS' CIRCLE**
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## Parity problems

(1) The product of 30 integers is equal to 1. Can their sum be equal to zero?
(2) Can an ordinary $8 \times 8$ chessboard be covered with $1 \times 2$ dominoes so that only two corner squares on the same diagonal remain uncovered?
(3) A box is filled with 75 white beads and 150 black ones. There is a pile of black beads near the box. Remove two beads from the box. If one is black, put back the other (white or black). If both are white, put in a black one from your pile. Each time one repeats this process, there will be one less bead in the box. What will be the color of the final bead left in the box?
(4) Of 101 coins, 50 are counterfeit, and they differ from the genuine coins in weight by 1 gram. Peter has a scale in the form of a balance which shows the difference in weight between the objects placed on each pan. He chooses one coin, and wants to find out whether it is counterfeit. Can he do this in one weighing?
(5) Can you find any integer solutions to the equation $5x(x+1) = 3(2y+1)^3$?

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Given 5 lattice points in the plane, prove that there are two of them whose midpoint is a lattice point. (A lattice point is a point both of whose coordinates are integers.)

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## The pigeon-hole principle

(1) If 9 people are seated in a row of 12 chairs, then some consecutive set of 3 chairs are filled with people.
(2) At any party some pair of people are friends with the same number of people there.
(3) Given any 6 points inside a circle of radius 1, some two are within 1 of each other.
(4) If 11 of the first 20 integers are selected, then then some two of the selected integers have the property that one divides the other.
(5) Suppose each point of the plane is colored red or blue. Show that some rectangle has its vertices all the same color.
(6) Is there a multiple of 2015 that is of the form $5555\ldots5$, i.e. a number with all the digits being 5?