



SYMMETRY EXPLORATION

Sam Vandervelde
samv@mandelbrot.org
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Problems.

1. We decided today that there were exactly 8 symmetries of the square. Describe them all by drawing a square with vertices A , B , C , and D and then showing where the vertices go after applying each type of symmetry. (For example, we could rotate the square by 90° .)
2. Choose one rotational symmetry and one mirror symmetry from your list in the previous problem. Apply them one after the other and describe the resulting symmetry. Now try combining them in the other order. Do you obtain the same result?
3. Find a shape in the plane that has exactly five symmetries.
4. Build a Zometool cube from twelve blue struts and eight white nodes.
5. Recall that, for our purposes, a symmetry of the cube is a rotation of the cube about some axis which returns the cube to its original position in space. (But the order of the vertices and edges might be changed.) Predict the number of symmetries by counting the number of places a given face of the cube can wind up. (HINT: break this process up into two steps—first count the number of faces available, then count the number of ways the given face can be matched up with that face.)
6. Now confirm your prediction from the previous problem by actually describing all the symmetries of the cube. To begin, there are three different sorts of axes of symmetry. What are they? And how many of each type are there? (For example, one such axis is the type which passes through the center of two opposite faces. There are three of these axes; one for each pair of opposite faces.)
7. For each axis of symmetry, how many symmetries can be obtained by rotating the cube about this axis by some non-zero angle? (For example, given an axis through the centers of a pair of opposite faces, there are three possible rotations: by 90° , 180° , and 270° .) Now see if everything adds up. Don't forget to include the identity symmetry!